# **Appendix: 18 Example-Tracing Tutors**

We provide screenshots of the 18 example-tracing tutors built since 2009 that are discussed in the body of the document.

#### Mathtutor

		1234	56	78910			k 🗙				
Twenty people ar more children tha	e going to a concert. Th n adults.	ere are eight				Ţ					
How many childre	en and adults were at th	e concert?	chile	children adults							
			adu								
			ndeed! So how n	iany peo	ple is ONE unknown part?						
Ν	lumber of children =			2 x			<b>=</b> 12				
	Number of adults =			1 x			= <mark>12/2</mark>				
Peop Hint	ro parts together are 12 ole would one part be?	people, how many				Find Sum of Parts Identify Given Values Identify Unknown Part Interpret Representation Set-up Equation	s				
-	Previous Next 🔶					Solve Equation					
							li.				
You are walking ir you are going. Yoi	a a forest holding a GPS o u try to walk at a contan	1 2 3 4 device that tells you ho t pace of 3 miles per h	56 ow fast our.	7 8 9 10 Enter the y-co line. You can u	ordinate se the p	s in the table, then plot the oints to read-off the x-value	points on the s.				
<ol> <li>What distance will</li> <li>How far will you h</li> <li>What distance will</li> </ol>	l you have walked in two ho ave walked if you walk for t I you have walked in four h	urs? hree hours? purs?		30	<b>□</b>    ∱h <sup>3</sup> °	x-axis scale 1 y-a	xis scale 1				
Now, use the line 4. How long does it	you have drawn to ans take to walk 15 miles?	ver 4 and 5 below.			25						
5. The next town is	27 miles away. How long wi	ll it take you to reach it?		distance	15						
Quantity Name Unit	time hours	miles	_		10						
Question 1 Question 2	3	6 9			5						
Question 4	5 please place a point first	15		0	0	1 2 3 4 5	678 58				
Equation	у =					_					
<b>?</b> Hint The pright axis	point for question 5 woul now. Please adjust the (in the bottom right box)	ld not fit on the graphe right boundary of the >	er K-			Calculate y value Determine x coordinate Draw a line Find equation	Done				
L Instructions	revious Next 🔶					Name quantities Name units Dist point given coords	T				

*Mathtutor* (Aleven et al. 2009a) is a comprehensive web-based tutoring system for mathematics in grades 6 through 8.

#### **Genetics** Tutor



The *Genetics Tutor* (Corbett et al. 2010) covers a wide range of problem-solving activities in high-school and college-level genetics

	5x+2	= 2x+8							
	5x+2-2	= 2x+8-	2	subtract	[?] from both sides	• 2			
	5x	= 2x+6							
	5x-2x	= 2x+6-2	x	subtract	[?] from both sides	▼ 2x			
	3x	= 6							
	3x/6	=							
<b>P</b> int	What can you	do to both sides	to get x by i	tself?			So	lution: x =	D
ictions	Previous	Next 🔶							
actions	← Previous	Next 🔶	_						
ections	Previous	Next 🔶			•				
n	+ Previous	Next 🔸			•				×
ictions	Previous	Next →	t unfortunately, y	ou are back to -73	• = 9 - 4x, the same				×
ections	Previous	Next	t unfortunately, y	ou are back to -73 ething other than 1	= 9 - 4x , the same		Hint		×
ctions N Ho H	Previous  Mathematically, we equation as before      Previous	Next	t unfortunately, y or subtract some	ou are back to -7) thing other than '	● = 9 - 4x, the same 5.		Hint		×
ictions Ho H R	Previous      Mathematically, we equation as before     ends of the previous      please solve fit	Next	t unfortunately, y or subtract some	ou are back to -7, ething other than ! - 4x	• = 9 - 4x, the same 3,	*	Hint Undo	Done	×
ictions N Ho R	Previous  Mathematically, w equation as before      Previous  please solve f	Next    Next	t unfortunately, y or subtract some = -1	ou are back to -7) thing other than ' - 4x	• = 9 - 4x, the same Next	•	Hint Undo	Done	×
Ho Ho R	Previous  Mathematically, w equation as before      Previous  please solve f	Next   Next  A you did is correct, bu To make progress, add  or X: -10 - 7x  -7x-10+10	t unfortunately, y or subtract some = -1	ou are back to -7) thing other than - <b>- 4</b> x	• = 9 - 4x, the same	→ 7 10	Hint Undo	Done th sider *	×
Ho H	Previous      Mathematically, we equation as before     equation as before     Previous      please solve f	Next   Next  At you did is correct, bu  to make progress, add  or X: -10 - 7x  -7x-10+10  -7x	t unfortunately, y or subtract some = -1 = -1 = -1	ou are back to -7. ething other than the second sec	• = 9 - 4x, the same Next	- 10	Hint Undo to/from bo	Done th sider *	×
ections He H R	Previous      Mathematically, we equation as before     Previous      please solve f	Next → at you did is correct, bu r, To make progress, add or x: -10 - 7x -7x-10+10 -7x -7x-9	t unfortunately, y or subtract some = -1 = -11 = 9-4	ou are back to -77 ething other than to - <b>4</b> x *10-4x fx	• • • • • • • • • • • • • • • • •	- 10	Hint Undo to/from bo	Done th sider *	X
R R	Previous      Mathematically, we equation as before     erevious      please solve for	Next → at you did is correct, bu To make progress, add or x: -10 - 7x -7x-10+10 -7x -7x-9 -7x	t unfortunately, y or subtract some = -1 = -1 = -1 = -1 = -1 = -1 = -1 = -1	ou are back to -77 sthing other than to - 4x > 10-4x fix fix fix	= 9 - 4x, the same . Next Added Subtracted Added	→ 10 • 9 • 9	Hint Undo to/from bo	Done th aider • th aider •	×
Ho H	Previous	Next → at you did is correct, bu To make progress, add or x: -10 - 7x -7x -10+10 -7x -7x -9 -7x -7x	t unfortunately, y or subtract some = -1 = -1 = -4 = -4 = -4	ou are back to -77- sthing other than to - <b>4</b> x > 10-4x & & & & & & & & & & & & & & & & & & &	<ul> <li>a 9 - 4x, the same</li> <li>b.</li> <li>Next</li> <li>Added</li> <li>Subtracted</li> <li>Added</li> </ul>	→ 10 × 9 × 9	Hint Undo to/from bo	Done th aider * th aider *	×
in the	Previous	Next → at you did is correct, bu To make progress, add or x: -10 - 7x -7x -10+10 -7x	tunfortunately, y or subtract some = -1 = -1 = -40 = -40 = -40	ou are back to -7- sthing other than to - 4x > 10-4x & & & & & & & & & & & & & & & & & & &	<ul> <li>Added</li> <li>Subtracted</li> <li>Added</li> </ul>	→ 10 × 9 × 9	Hint Undo to/from bo	Done th aider * th aider * th aider *	×
He He He R	Previous	Next → at you did is correct, bu To make progress, add or x: -10 - 7x -7x -10+10 -7x -7x -9 -7x -7x -7x -7x -7x -7x -7x -7x -7x	t unfortunately, y or subtract some = -1 = -1 = -4 = -4 = -4 = -4 = -4	ou are back to -7- sthing other than to - 4x e10-4x & & & & & & & & & & & & & & & & & & &	<ul> <li>Added</li> <li>Subtracted</li> <li>Added</li> </ul>	→ 10 ↓ 9 ↓	Hint Undo to/from bo	b aider * th aider * th aider *	×
R R H R	Previous	Next → at you did is correct. bu To make progress, add or x: -10 - 7x -7x -0 +10 -7x -7x -2 -7x -2 -7x	tunfortunately, y or subtract some = -1 = -1 = -4 = -4 = -4 = -4 = -4 = -4 = -4 = -4	ou are back to -7- thing other than to - 4x +10-4x k k k k k k k k k k k k k k k k k k k	<ul> <li>Added</li> <li>Subtracted</li> <li>Added</li> <li>G</li> </ul>		Hint Undo to/from bo	b aider * th aider * th aider *	×
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ctors	Previous      Mathematically, w     equation as before     errors      Previous      please solve f      y      B      e      Previous      y	Next         →           at you did is correct, bu	tunfortunately, y or subtract some = -1 = -1 = -1 = -1 = -1 = -1 = -1 = -1	ou are back to -7- thing other than the second seco	<ul> <li>Added</li> <li>Subtracted</li> <li>Added</li> <li>G</li> <li>G</li> <li>G</li> <li>A</li> </ul>		Hint       Undo       to/from bol       to/from bol       to/from bol       1       7       -       0	th aides - th ai	)

# Lynnette – Basic Equation Solving

*Lynnette* is a tutor for basic equation solving for grades 6, 7, and 8, was originally implemented as an exampletracing version (top) (Long and Aleven 2013a, b; Waalkens et al. 2013), and was later re-implemented, also with CTAT, as a rule-based Cognitive Tutor (bottom) (Long and Aleven 2014)

# The Tuning Tutor - Parameter Fitting in Machine Learning

			Paramet	ter Tunin	g: 🖣 1 <b>2</b>	3456	67891	0 🕨			
In the first stage we selected the setting we would use to build a tuned model over the whole data set (which we would do in stage 2). Now we want to estimate what that model's performance would be on new data (stage 3). We do that using an embedded cross validation.											
In the table be outer loop. Fo validation. Th inner loop, ins hold out. The the validation fold, which we	elow, the t or the oute ie only exc tead of a o hold out is set is wha e will then	op 5 rows reption her cross valid s the test t we test o use in the	represent just divide re is that w ation on th set, which on. Based outer loop	the inner each fold we may sel the training we will no on this tra	loop of the into a trai ect a differ data for e t consult o hin/test spl	cross vali ning set ar rent setting ach fold, v in the inne it for a fol	idation and t nd a testing g on each fo we divide ea er loop. The ld, we will m	the bottom 5 ro set, as we did Id based on the ch fold into trai train set is what ake a selection	ws represent the for the simple cross a inner loop. For th n, validation, and at we train on and of a setting for tha		
Training Validation Hold out A B C D We compare the baseline model with the tuned model to see whether the tuning process makes											
{Y3,Y4,Y5} {Y2} {Y1} 0.58 0.73 0.94							significant	improvement.	Tuned Medel		
{Y1,Y3,Y5}	{Y4}	{Y2}	0.45	0.59	0.52	0.67	Fold	Performance	Performance		
{Y1,Y2,Y4}	{Y5}	{Y3}	0.74	0.67	0.56	0.68	{Y1}	0.59	0.62		
{Y1,Y2,Y5}	{Y3}	{Y4}	0.78	0.79	0.67	0.5	{Y2}	0.61	0.82		
{Y2,Y3,Y4}	{Y1}	{Y5}	0.69	0.75	0.76	0.43	{13}	0.71			
{Y2,Y3,Y4,Y5}	{Y1}	•	0.59	0.58	0.62	0.57	{Y5}	0.59			
{Y1,Y3,Y4,Y5}	{Y2}	•	0.61	0.69	0.62	0.82	Average	0.662			
{Y1,Y2,Y4,Y5}	{Y3}	-	0.81	0.37	0.79	0.76	1. Please c	lick on the highest e inner loop and th	performance value for ten click on the		
{Y1,Y2,Y3,Y5}	{Y4}	•	0.71	0.8	0.53	0.57	correspond	ing performance v	alue for the outer loop.		
{Y1,Y2,Y3,Y4}	{Y5}	· · ·	0.59	0.68	0.52	0.64					
The performance highest performa	e value you h nce value.	ave selected	I on the inner	r loop is not t	he	? Hint	_				

The *Tuning Tutor* helps graduate students and advanced undergraduate students learn to use cross validation to avoid overfitting when tuning model parameters. It was used at Carnegie Mellon University in a course for graduate students and advanced undergrads called "Applied Machine Learning" by Carolyn Rosé

# **Stoichiometry Tutor**

Stoichiometry Tutor   Worked Example	Help								
Problem Statement									
Let's convert a substance that is in milligrams to grams. We'll calculate the number of grams (g) that are in 10.6 milligrams (mg) of wood alcohol (COH4). Our result should have 3 significant figures.									
Problem         Units         Substance         #         Units         Substance         #           10.6         mgconst_v         1         g < COHH	#     Units     Substance      #     Units     Substance       Image: Substance      #     Units     Substance     #       Image: Substance      Image: Substance      #     Units     Substance       Image: Substance      Image: Substance      #     Units     Substance       Image: Substance      Image: Substance      Image: Substance      #     Units       Image: Substance      Image: Substance      Image: Substance      Image: Substance        Image: Substance      Image: Substance      Image: Substance      Image: Substance </td								
Stoichiometry Tutor   Worked Example	Help								
Problem Statement									
Let's convert a substance that is in milligrams to grams. We'll of that are in 10.6 milligrams (mg) of wood alcohol (COH4). Our refigures.	calculate the number of grams (g) suit should have 3 significant								
Problem	Result								
# Units Substance - # Units Substance -	Units Substance / # Units Substance / # Units Substance     Units Substance / # Units Substance								
Unit Conversion   Unit Conversion	mas v mas v								
There are some errors in the solution. The step you are ready, select the 'Next' button to move	is in red are incorrect. Please take some time to review your work. When on.								
	Next								

The *Stoichiometry Tutor* (McLaren et al. 2014, 2015b, 2016) supports the narrated replay of example solutions. As the steps of the problem are replayed, a flashing yellow box draws the student's attention to the next step of the worked example (top). After the worked example plays back, the student is prompted to fill out the reasons for every step, and then their solution is evaluated (i.e., delayed feedback; bottom).

# AdaptErrEx – Erroneous Examples

Allison has two ribbons. One ribbon is 0.125 inches long and the other is 0.8	33 inches long. Allison's friend asks her to choose the longest ribbon.
Allison said this incorrect answer: I placed the numbers on a number line to see that 0.125 is the largest, so I want the 0.125 inch long ribbon.	What did Allison do wrong?         She thinks that         Ionger decimals are smaller         shorter decimals are smaller than zero         shorter decimals are larger         Ionger decimals are larger
Click on the line where the incorrect point should go to fix Allison's error.	Looking at the corrected number line, which ribbon is longer?   O.83 inches because it is closer to 1  O.125 inches because it is closer to 0
What advice would you give to Allison to solve the problem right next time? Allison, to find the longest ribbon you should pay attention to which decima is the shortest is the longest has the smallest number in the tenths place in the largest number in the tenths place	Message Window Nou've got it. Well done.
	Done

AdaptErrEx (Adams et al., 2014; McLaren et al. 2015) is an example-tracing tutor for learning decimals (part of 6th-grade mathematics) that has students identify, correct, and explain incorrect steps in worked-out problem solutions



**Decimal Point: Educational Games for Learning Decimals** 

Decimal Point (Forlizzi et al., 2014), built using CTAT as foundation, supports game-based learning with erroneous examples to help middle-school students learn decimals

# **Proportional Reasoning Tutor**

	Practice Problems: Page 6 of 17	
3. Bill's Hometown F identical kitchen cab weekend, one of the complete the remain	urniture Store creates custom-ordered furniture. Last week, Bill, the owner, received an order for 12 inets. Bill hired 4 carpenters who were able to make 7 cabinets in 5 days. Unfortunately, over the carpenters broke his arm and will be unable to help finish the order. If Bill has 3 healthy carpenters ing cabinets, how long will it take them to finish the job?	2
Step 1.		
4		
	A	
Step 2.		
4	$\cdot \mathbf{x} = 7 \cdot 3$	
4x	= 21	
Step 3.		
	4	
x		
x	= 0	
Diagon entre	1941 in the binklinks of Fold	
?	21 in the highlighted held.	
Hint	Next ->	

The Proportional Reasoning Tutor (Earnshaw, 2014) supports worked examples and tutored problems in middle-school mathematics

# **Fractions Tutor**

![](_page_8_Figure_2.jpeg)

![](_page_8_Figure_3.jpeg)

The *Fractions Tutor* (Rau et al. 2013, 2014, 2015a) supports conceptual learning of fractions in grades 4 and 5 using multiple interactive graphical representations of fractions

## **Grounded Feedback Tutor**

![](_page_9_Figure_2.jpeg)

*Grounded Feedback Tutor* (Stampfer & Koedinger, 2013; Wiese & Koedinger, 2015) for elementary school fractions learning uses a graphical representation to provide feedback on students' solutions, instead of providing explicit correctness feedback. As the student enters a solution using numeric symbols, the fraction bars (except those representing the given fractions) are updated by the system to reflect the student input

← Previous Next →

![](_page_10_Figure_1.jpeg)

# **Chem Tutor**

In *Chem Tutor* (Rau 2015; Rau and Wu 2015; Rau et al. 2015a, b), designed for introductory undergraduate chemistry learning, students plan and construct a graphical representation (Lewis structures), with feedback from the system

![](_page_10_Figure_4.jpeg)

Given one representation, students construct a different kind of graphical representation of the same atom and are prompted to reflect on the differences and limitations of the two visual representations

![](_page_11_Figure_2.jpeg)

The *RedBlackTree Tutor* (Liew & Xhakaj, 2015; Xhakaj, 2015; Xhakaj & Liew, 2015) helps students learn an algorithm for building red-black trees, a common data structure in computer science

#### **Equivalent Fractions** A Match the equivalent fractions. ? Match an equivalent fraction for each fraction shown below. Make sure the circles show the same amounts before Hint 1 hitting submit. You and your partner can each move only half of the fractions. Discuss with your partner what the correct answers are 8 5 2 6 16 1 10 9 7 7 24 4 3 3 - Previous Next 9 12 6 14 4 7 Submit

# Tutor for collaborative learning of fractions

![](_page_12_Figure_3.jpeg)

Elementary school students (grades 4 and 5) use the *Collaborative Fractions Tutor* with a partner; each partner has a different role, with a different view of the problem and different available actions (Olsen et al. 2014a, b, under review)

# Tutor for Business Modeling with Google Sheets

⊞	Alex Proces File Edit Vie	sing - Tutored w Inset Forma	f Problem It Data Tools	IIII Add-ons Help	CTAT Last ed	It was made seconds	ago by Bruce N	IcLaren					Comments () Share
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fx -													
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7	Output												
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	1997	3.91	4	5	2	9	1	2	5				
10	1998	5.06	4	5	2	10	1	3	6				In the next step you will want to
11	1999	6.05	4	4	3	11	1	4	7				calculate the productivity of other
12	2000	7.77	5	7	3	13	2	5	8				machine types by copying this cell to
13	2001	10.03	6	8	4	15	2	5					other cells, so you should fix the
54	2002	12.01	7	55	4	16	3	7	10				aggregate demand column by using a
15	2003	14.61	7	12	5	19	3	7	11				3.
16	2004	17.39	8	15	6	20	3		12				
17	2005	20.82	9	18	6	23	5	9	14				Previous
18	2006	23.92	10	19	7	24	5	10	14				
19	2007	27.34											
20	2008	31.9											•
21	2009	36.04											· · · · · · · · · · · · · · · · · · ·
22	2010	40.73											Hint Done
23	2011	45.22											
24	2012	50.19											
25	2013	55.36											
26	2014	62.02											
27	2015	68.03											
28	2016	73.29											
29	0.000												
30	Year	ggregate Dema	an Grinder Machin	e Grader Machine	Selve Machine 3	iorting Equipmentilione	ing Machine Pa	oking Mach	nine Test Equipment				
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32	1997	3.91											
33	1998	5.06											
ж	1999	6.05											
35	2000	7.77	15										8

An example-tracing tutor build by McLaren and colleagues, embedded within Google Sheets, provides guidance with business modeling problems

#### Fractions Tutor version that supports Sense Making, Induction/Refinement, and Fluency Building

A new version of the Fractions Tutor (Doroudi et al., 2015) has activities targeting each of the three main learning mechanisms identified in the Knowledge-Learning-Instruction framework (KLI; Koedinger et al., 2012) induction and refinement (IR) (top), sense-making (SM) (middle), and fluency (F) (bottom)

![](_page_14_Figure_3.jpeg)

![](_page_15_Figure_1.jpeg)

![](_page_15_Figure_2.jpeg)

A tutor by Roll et al. (2010) provides guidance during invention activities

![](_page_15_Figure_4.jpeg)

CTAT was used to create a tutor for guided invention activities with a Wizard of Oz interface (Chase et al., 2015); CTAT s collaborative tutoring facility enabled separate roles and capabilities for student and wizard.

#### The Article Tutor

![](_page_16_Picture_2.jpeg)

Version of the Article Tutor (Wylie et al. 2011) that supports self-explanation